

Practice Questions Solutions

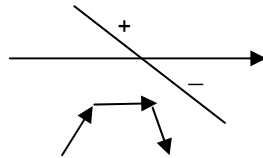
Curve Sketching

$$(a) (i) \frac{d}{dx} \left(4x^{\frac{1}{4}} - x^{\frac{5}{4}} \right) = x^{-\frac{3}{4}} - \frac{5}{4}x^{\frac{1}{4}} = \frac{4-5x}{4x^{\frac{3}{4}}}.$$

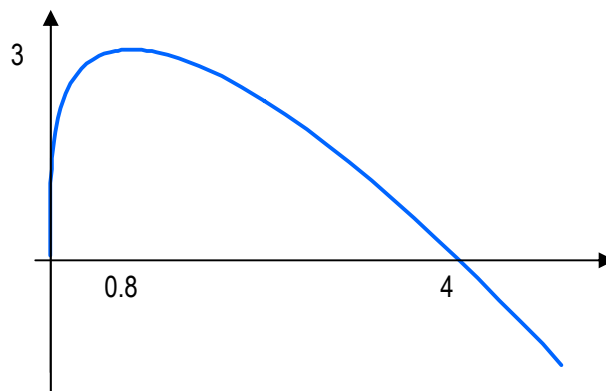
\therefore As $x \rightarrow 0^+$, $f'(x) \rightarrow \infty$, the curve has a vertical tangent.

$$(ii) f'(x) = 0 \text{ gives } x = \frac{4}{5} \therefore \text{Turning point } \left(\frac{4}{5}, \frac{16}{5} \sqrt[4]{\frac{4}{5}} \right) \approx \left(\frac{4}{5}, 3 \right).$$

As $x^{\frac{3}{4}} > 0$, the sign of $f'(x)$ is decided by $4-5x$, \therefore this is a maximum point.



(iii) For the x -intercepts, let $y = 0$, $x = 0$ or 4 .



(iv) Without using Calculus, sketch the following curves

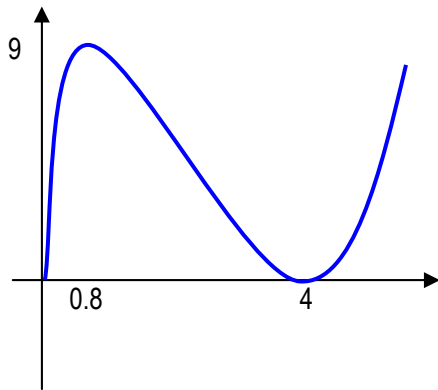
(α) $y = f^2(x)$.

(β) $y^2 = f(x)$.

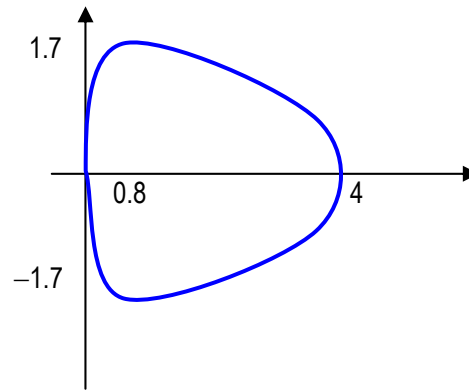
(γ) $y = \ln(f(x))$.

(δ) $y = e^{-f(x)}$.

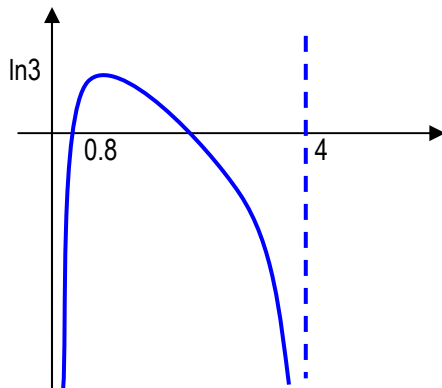
(α)



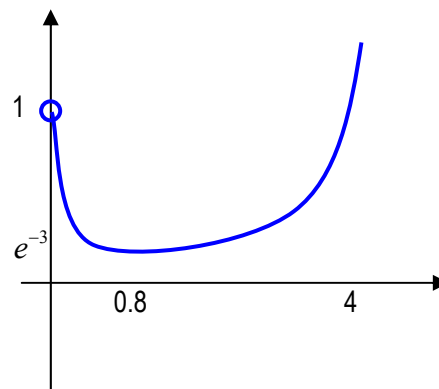
(β)



(γ)



(δ)



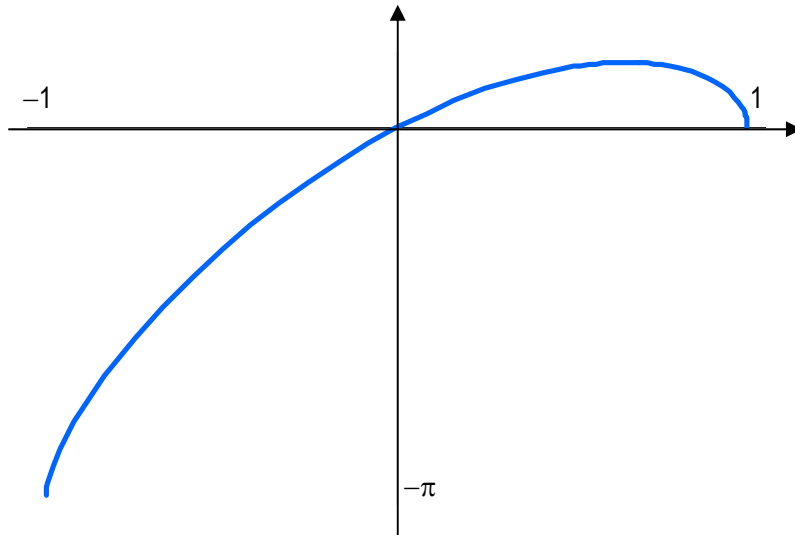
(b) (i) $f'(x) = \cos^{-1} x - \frac{x}{\sqrt{1-x^2}}$

$$f''(x) = -\frac{1}{\sqrt{1-x^2}} - \frac{\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}}}{1-x^2} = -\frac{1}{\sqrt{1-x^2}} - \frac{1-x^2+x^2}{(1-x^2)\sqrt{1-x^2}}$$

$$= -\frac{1-x^2}{\sqrt{(1-x^2)^3}} - \frac{1}{\sqrt{(1-x^2)^3}} = \frac{x^2-2}{\sqrt{(1-x^2)^3}}$$

(ii) For $-1 \leq x \leq 1$, $\sqrt{(1-x^2)^3} > 0$ but $x^2 - 2 < 0$, $\therefore f''(x) < 0$: the curve is concave downwards.

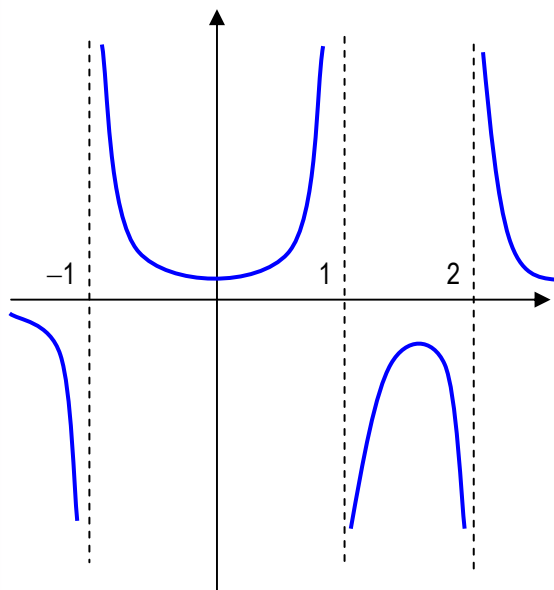
(iii)



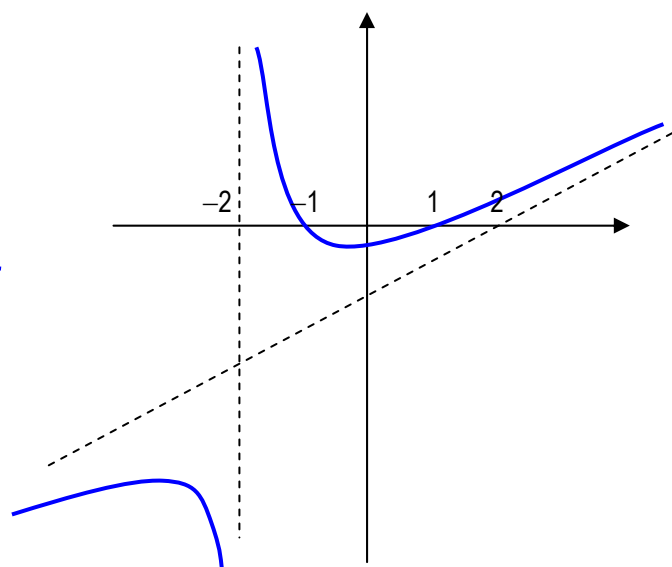
(c) (i) $y = \frac{1}{(x-2)(x^2-1)}$

(ii) $y = \frac{x^2-1}{x+2} = x-2 + \frac{3}{x+2}$

(i)



(ii)

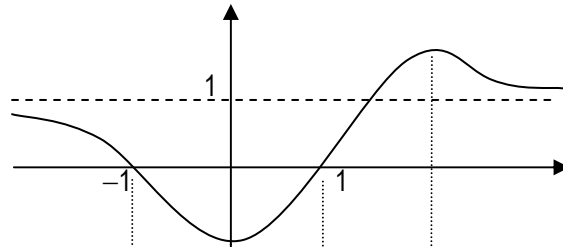


(d) Given the graph of $y = f'(x)$ as shown.

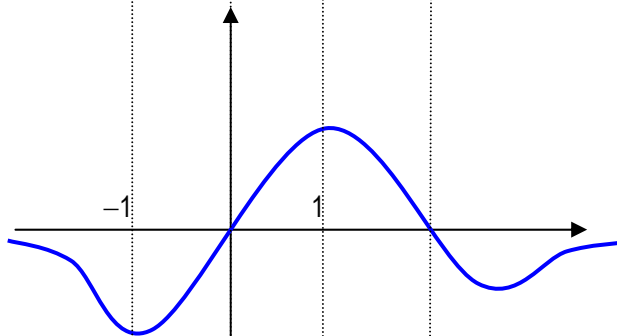
Sketch

(i) $y = f''(x)$.

(ii) $y = f(x)$, given $f(1) = 0$.



(i)



(ii) (The asymptote is $y = x + C$)

