Question 1

(a) $\int x\ln x dx.$

Let $u=\ln x, du=\frac{1}{x}dx$ and $dv=xdx, v=\frac{x^2}{2}.$

$\therefore I=\frac{x^2}{2}\ln x-\int\frac{x}{2}\,dx=\frac{x^2}{2}\ln x-\frac{x^2}{4}+C.$

(b) $\int_0^3 x\sqrt{x+1}\,dx=\int_0^3 (x+1-1)\sqrt{x+1}\,dx$

$=\int_0^3\sqrt{(x+1)^3}\,dx-\int_0^3\sqrt{x+1}\,dx$

$=\left[\frac{2\sqrt{(x+1)^5}}{5}-\frac{2\sqrt{(x+1)^3}}{3}\right]_0^3$

$=\left(\frac{2}{5}\times 32-\frac{2}{3}\times 8\right)-\left(\frac{2}{5}-\frac{2}{3}\right)=\frac{116}{15}.$

(c) $\frac{1}{x^2(x-1)}=\frac{1}{x-1}+\frac{-x-1}{x^2}=\frac{1}{x-1}-\frac{1}{x}-\frac{1}{x^2}.$

$\therefore c=1, b=-1, a=-1.$

$\therefore\int\frac{1}{x^2(x-1)}dx=\ln(x-1)-\ln x+\frac{1}{x}+C.$

(d) $\int\cos^3\theta\,d\theta=\int(1-\sin^2\theta)\cos\theta\,d\theta$

$=\sin\theta-\frac{\sin^3\theta}{3}+C.$

(e) $\int_{-1}^1\frac{1}{5-2t+t^2}dt=\int_{-1}^1\frac{1}{(t-1)^2+4}dt$

$=\frac{1}{2}\left[\tan^{-1}\frac{t-1}{2}\right]_{-1}^1=\frac{1}{2}\left(0-\tan^{-1}(-1)\right)$

$=\frac{1}{2}\tan^{-1}1=\frac{\pi}{8}.$

Question 2

(a) (i) $\overline{w}+z=2+3i+3+4i=5+7i$

(ii) $|w|=\sqrt{2^2+3^2}=\sqrt{13}$

(iii) $\frac{w}{z}=\frac{2-3i}{3+4i}=\frac{2-3i}{3+4i}\times\frac{3-4i}{3-4i}$

$=\frac{6-12-9i-8i}{25}=\frac{-6-17i}{25}.$

(b) (i) $z=\left(\sqrt{3}+i\right)+\left(1+i\sqrt{3}\right)=\left(1+\sqrt{3}\right)+i\left(1+\sqrt{3}\right)$

(ii) Let $A=\sqrt{3}+i, B=1+i\sqrt{3},$

$\angle BOA=\tan^{-1}\sqrt{3}-\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}.$

$\therefore\theta=\pi-\frac{\pi}{6}$ (cointerior angles on parallel lines)

$\therefore\theta=\frac{5\pi}{6}.$

(c) $z^3=8=8\text{cis}(2k\pi).$

$\therefore z=2\text{cis}\frac{2k\pi}{3}, k=0,\pm1$

$=2\text{cis}0, 2\text{cis}\frac{2\pi}{3}, 2\text{cis}\left(-\frac{2\pi}{3}\right).$

(d) (i) $(\cos\theta+i\sin\theta)^3=c^3+3c^2(is)+3c(is)^2+(is)^3,$

where $c=\cos\theta, s=\sin\theta,$

$=\cos^3\theta+i3\cos^2\theta\sin\theta-3\cos\theta\sin^2\theta-i\sin^3\theta$

(ii) $(\cos\theta+i\sin\theta)^3=\cos3\theta+i\sin3\theta.$

$\therefore\cos3\theta=\cos^3\theta-3\cos\theta\sin^2\theta,$ equating the real parts

$=\cos^3\theta-3\cos\theta(1-\cos^2\theta)$

$=4\cos^3\theta-3\cos\theta.$

$\therefore 4\cos^3\theta=\cos3\theta+3\cos\theta.$

$\therefore\cos^3\theta=\frac{1}{4}\cos3\theta+\frac{3}{4}\cos\theta.$

(iii) $4\cos^3\theta-3\cos\theta=\cos3\theta=1.$

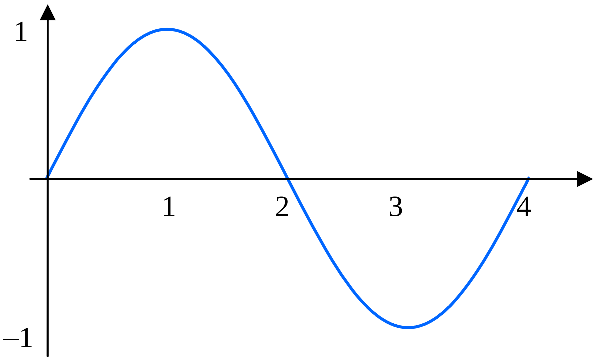
$\therefore 3\theta=2k\pi, k\in Z.$

$\therefore\theta=\frac{2k\pi}{3}.$

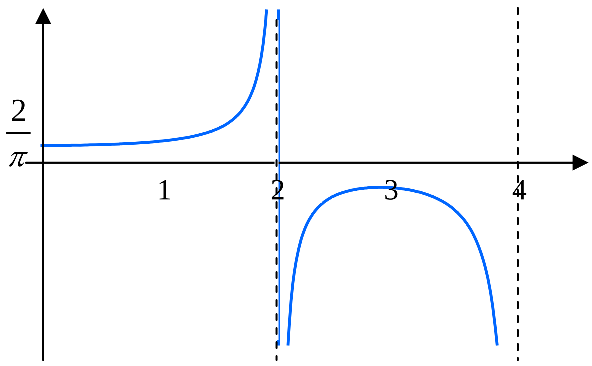
$\therefore$ The smallest positive solution is $\theta=\frac{2\pi}{3}.$

Question 3

(a) (i) Amplitude $=1$, Period $=4$



(ii) $\lim_{x\to 0}\dfrac{x}{\sin\frac{\pi x}{2}}=\lim_{x\to 0}\dfrac{\frac{\pi x}{2}}{\sin\frac{\pi x}{2}}\times\dfrac{2}{\pi}=\dfrac{2}{\pi}.$



(b) Cross-section has base $2y$, height $\sqrt{1-y^2}$

$\therefore A=\frac{1}{2}\times 2y\times\sqrt{1-y^2}=y\sqrt{1-y^2}.$

$\partial V=y\sqrt{1-y^2}\times\partial x=\cos x\sqrt{1-\cos^2 x}\,\partial x$

$=\cos x\sin x\,\partial x.$

$\therefore V=\int_0^{\frac{\pi}{2}}\cos x\sin x\,dx=\frac{1}{2}\left[\sin^2 x\right]_0^{\frac{\pi}{2}}=\frac{1}{2}\text{ u}^3.$

(c) Let $n=1$, LHS $=2$, RHS $=2\times 1^2=2$.

$\therefore$ True for $n=1$.

Assume $(2n)!\geq 2^n(n!)^2$,

RTP $(2(n+1))!\geq 2^{n+1}((n+1)!)^2$.

LHS $=(2n)!(2n+1)(2n+2)\geq 2^n(n!)^2(2n+1)(2n+2)$

$>2^n(n!)^2(n+1)2(n+1)$, since $2n+1>n+1$,

$\geq 2^{n+1}(n!)^2(n+1)^2$

$=2^{n+1}((n+1)!)^2$.

$\therefore$ True for $n+1$.

$\therefore$ True for all $n\geq 1$ by the principle of Induction.

(d) (i) $9=16(e^2-1)$, using $b^2=a^2(e^2-1)$

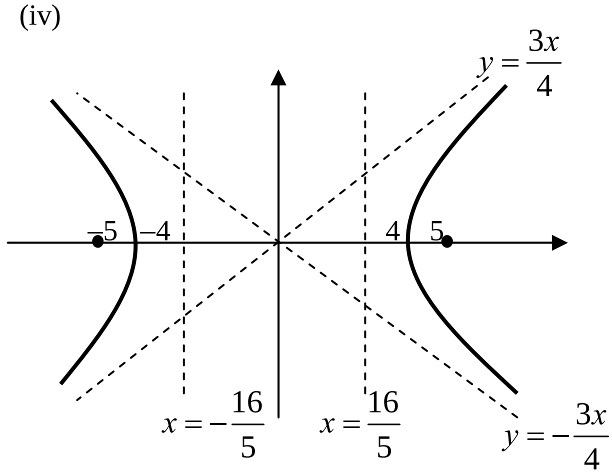
$\therefore e^2=\frac{9}{16}+1=\frac{25}{16}.$

$\therefore e=\frac{5}{4}.$

(ii) Foci $(\pm 5,0)$

(iii) Asymtptotes $y=\pm\frac{3x}{4}$.

(iv)



(v) $e^2=1+\frac{b^2}{a^2}$, $\therefore$ when $e\to\infty$, either $a\to 0$ or $b\to\infty$: the hyperbola flats out and becomes the y-axis or parallel to the y-axis.

Question4

(a) (i) $|z-a|^2 - |z-b|^2 = 1$

$(x-a)^2 + y^2 - \left((x-b)^2 + y^2\right) = 1$

$x^2 - 2ax + a^2 + y^2 - \left(x^2 - 2bx + b^2 + y^2\right) = 1$

$2(b-a)x + a^2 - b^2 = 1$

$2(b-a)x = 1 - a^2 + b^2.$

$x = \dfrac{1}{2(b-a)} + \dfrac{b^2 - a^2}{2(b-a)}$

$= \dfrac{1}{2(b-a)} + \dfrac{a+b}{2}.$

(ii) The locus of z is the vertical line of equation

$x = \dfrac{1}{2(b-a)} + \dfrac{a+b}{2}.$

(b) (i) $\angle ADG = \angle ABC$ (in a cyclic quad, the interior angle = the opposite exterior angle)

$\angle AFG = \pi - \angle ABC$ (cointerior angles on parallel lines)

$\therefore \angle ADG = \pi - \angle AFG.$

$\therefore AFGD$ is a cyclic quad (opposite angles are supplementary)

(ii) alternate angles on parallel lines

(iii) $\angle GAD = \angle GFD$ (angles subtending same arc)

$\therefore \angle GAD = \angle AED$ (because both $= \angle GFD$)

$\therefore GA$ is tangent to the circle $ABCD$ (angle between a tangent and a chord = angle in alternate segment)

(c) (i) Let $y = Af + Bg$, where f and g satisfy the given differential equation,

$\dfrac{dy}{dt} = A\dfrac{df}{dt} + B\dfrac{dg}{dt}.$

$\dfrac{d^2y}{dt^2} = A\dfrac{d^2f}{dt^2} + B\dfrac{d^2g}{dt^2}.$

Substituting to $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y$ gives

$A\dfrac{d^2f}{dt^2} + B\dfrac{d^2g}{dt^2} + 3A\dfrac{df}{dt} + 3B\dfrac{dg}{dt} + 2Af + 2Bg$

$= A\left(\dfrac{d^2f}{dt^2} + 3\dfrac{df}{dt} + 2f\right) + B\left(\dfrac{d^2g}{dt^2} + 3\dfrac{dg}{dt} + 2g\right)$

$= A\times 0 + B\times 0 = 0.$

$\therefore y = Af + Bg$ is also a solution.

(ii) If $y = e^{kt}$ is a solution,

$\dfrac{dy}{dx} = ke^{kt}, \dfrac{d^2y}{dx^2} = k^2e^{kt},$

$\therefore k^2e^{kt} + 3ke^{kt} + 2e^{kt} = 0.$

$e^{kt}\left(k^2 + 3k + 2\right) = 0.$

$k^2 + 3k + 2 = 0$, since $e^{kt} \neq 0.$

$(k+2)(k+1) = 0.$

$\therefore k = -2, -1.$

(iii) $y = Ae^{-2t} + Be^{-t}, \dfrac{dy}{dx} = -2Ae^{-2t} - Be^{-t}.$

When $t = 0, y = 0, \therefore 0 = A + B.$

When $t = 0, \dfrac{dy}{dx} = 1, \therefore 1 = -2A - B.$

Adding the two equations, $1 = -A.$

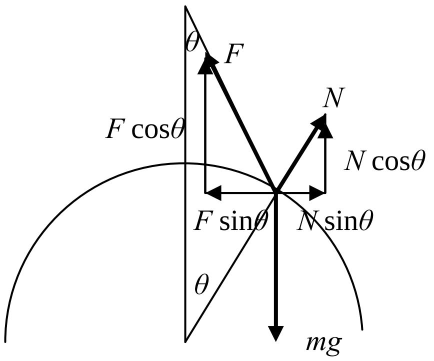
$\therefore A = -1, B = 1.$

Question 5

(a) (i) Resolving the forces,

horizontally, $\sum \text{force} = F\sin\theta - N\sin\theta =$ centripetal force $mr\omega^2$.

vertically, $\sum \text{force} = F\cos\theta + N\cos\theta - mg = 0,$ as there is no vertical motion.



(ii) $(F-N)\sin\theta = mr\omega^2, \therefore F - N = \dfrac{mr\omega^2}{\sin\theta}.$ (1)

$(F+N)\cos\theta = mg, \therefore F + N = \dfrac{mg}{\cos\theta}.$ (2)

$(2)-(1)$ gives $2N = \dfrac{mg}{\cos\theta} - \dfrac{mr\omega^2}{\sin\theta}.$

$\therefore N = \dfrac{1}{2}mg\sec\theta - \dfrac{1}{2}mr\omega^2\operatorname{cosec}\theta.$

(iii) The bead remains in contact with the sphere when $N \ge 0$.

$g\sec\theta - r\omega^2\operatorname{cosec}\theta \ge 0.$

$\omega^2 \le \dfrac{g\sec\theta}{r\operatorname{cosec}\theta} = \dfrac{g}{r}\tan\theta.$

But $\tan\theta = \dfrac{r}{h}, \therefore \omega^2 \le \dfrac{g}{h}, \therefore \omega \le \sqrt{\dfrac{g}{h}}.$

(b) $\dfrac{p}{1+p} + \dfrac{q}{1+q} - \dfrac{r}{1+r} = \dfrac{2pq+p+q}{1+p+q+pq} - \dfrac{r}{1+r}$

$= \dfrac{2pq+2pqr+p+rp+q+rq-r-rp-rq-pqr}{(1+p)(1+q)(1+r)}$

$= \dfrac{2pq+pqr+p+q-r}{(1+p)(1+q)(1+r)}$

$\ge \dfrac{2pq+pqr}{(1+p)(1+q)(1+r)},$ since $p+q \ge r$

$\ge 0.$

(c) (i) The reflection property of the ellipse:

When a light source is placed at a focus, the light ray from a focus, when hitting an elliptical mirror, will reflect through the other focus.

When light reflects at P, it reflects as if it moves from a light source at R, so that the distance travelled from S to P is the same as the distance travelled from R to P.

$\therefore PS = PR.$

$\therefore \Delta PQS \equiv \Delta PQR$ (RHS).

(Alternatively, let QPN be the tangent,

$\angle RPQ = \angle S'PN$ (vertically opposite angles)

$\angle S'PN = \angle SPQ$ (reflection angles)

$\therefore \angle RPQ = \angle SPQ.$

$\therefore \Delta PRQ \equiv \Delta PQS$ (AAS))

$\therefore SQ = RQ$ (corresponding sides in congruent triangles).

(ii) $SP + S'P = 2a$, by definition of the ellipse.

But $SP = PR$, $\therefore S'P + PR = S'R = 2a.$

(iii) O is the midpoint of $S'S$ and Q is the midpoint of RS, $\therefore OQ = \dfrac{1}{2}S'S$ (the join of the midpoints of two sides of a triangle is parallel to and equals half the third side).

$\therefore OQ = a.$

As O is fixed, the locus of Q is a circle of centre O, radius $a, \therefore Q \in x^2 + y^2 = a^2.$

Question 6

(a) (i) Terminal velocity occurs when acceleration = 0

$\therefore mg = kv^2.$

$\therefore v_T = \sqrt{\dfrac{mg}{k}}.$

(ii) $m\dfrac{dv}{dt} = mg - kv^2.$

$\therefore m\displaystyle\int \frac{dv}{mg-kv^2} = \int dt.$

$\therefore t = m\displaystyle\int \frac{dv}{\left(\sqrt{mg}-\sqrt{k}v\right)\left(\sqrt{mg}+\sqrt{k}v\right)}$

$= m \times \dfrac{1}{2\sqrt{mg}}\displaystyle\int \left(\frac{1}{\sqrt{mg}-\sqrt{k}v} + \frac{1}{\sqrt{mg}+\sqrt{k}v}\right)dv$

$= \dfrac{1}{2}\sqrt{\dfrac{m}{g}} \times \dfrac{1}{\sqrt{k}}\ln\dfrac{\sqrt{mg}+\sqrt{k}v}{\sqrt{mg}-\sqrt{k}v} + C$

$= \dfrac{1}{2g}\sqrt{\dfrac{mg}{k}}\ln\dfrac{\sqrt{mg}+\sqrt{k}v}{\sqrt{mg}-\sqrt{k}v} + C$

$= \dfrac{1}{2g}\sqrt{\dfrac{mg}{k}}\ln\dfrac{\sqrt{\dfrac{mg}{k}}+v}{\sqrt{\dfrac{mg}{k}}-v} + C$

$= \dfrac{v_T}{2g}\ln\dfrac{v_T+v}{v_T-v} + C$

When $t = 0, v = v_0, \therefore C = -\dfrac{v_T}{2g}\ln\dfrac{v_T+v_0}{v_T-v_0}.$

$\therefore t = \dfrac{v_T}{2g}\ln\dfrac{(v_T+v)(v_T-v_0)}{(v_T-v)(v_T+v_0)}.$

(iii) For Jac, $v_0 = \dfrac{1}{3}v_T, \therefore$ when his speed $= \dfrac{2}{3}v_T$,

the time taken is $\dfrac{v_T}{2g}\ln\dfrac{\left(v_T+\frac{2}{3}v_T\right)\left(v_T-\frac{1}{3}v_T\right)}{\left(v_T-\frac{2}{3}v_T\right)\left(v_T+\frac{1}{3}v_T\right)}$

$= \dfrac{v_T}{2g}\ln\dfrac{\frac{5}{3}\times\frac{2}{3}}{\frac{1}{3}\times\frac{4}{3}} = \dfrac{v_T}{2g}\ln\dfrac{5}{2}.$

For Gil, $v_0 = 3v_T, \therefore$ when her speed $= \dfrac{3}{2}v_T$,

the time taken is $\dfrac{v_T}{2g}\ln\dfrac{\left(v_T+\frac{3}{2}v_T\right)(v_T-3v_T)}{\left(v_T-\frac{3}{2}v_T\right)(v_T+3v_T)}$

$= \dfrac{v_T}{2g}\ln\dfrac{\frac{5}{2}\times -2}{-\frac{1}{2}\times 4} = \dfrac{v_T}{2g}\ln\dfrac{5}{2}.$

$\therefore$ The time taken for Jac's speed to double is equal to the time taken for Gil's speed to halve.

(b) (i) $y = (f(x))^3, y' = 3(f(x))^2 f'(x).$

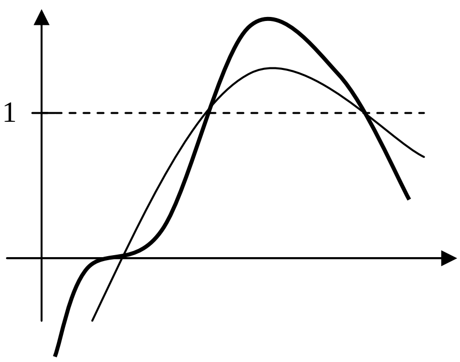
$\therefore$ If $f(a) = 0$ or $f'(a) = 0, y' = 0.$

$\therefore$ Stationary point at $x = a$ if $f(a) = 0$ or $f'(a) = 0.$

(ii) If $f(a) = 0$ and $f'(a) \neq 0$, then $y = 0, \therefore$ the stationary point occurs on the x-axis, $\therefore$ this is the triple root.

$\therefore$ It has a horizontal point of inflexion at $x = a$ if $f(a) = 0$ and $f'(a) \neq 0.$

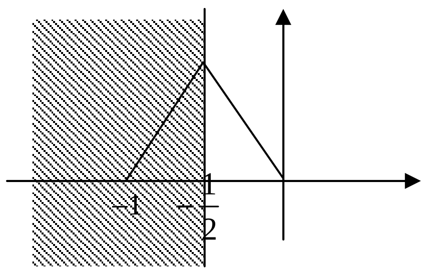
(iii)



(c) $\left|1+\dfrac{1}{z}\right| = \left|\dfrac{z+1}{z}\right| = \dfrac{|z+1|}{|z|}.$

$\therefore \left|1+\dfrac{1}{z}\right| \leq 1$ is equivalent to $|z+1| \leq |z|.$

$\therefore$ The locus of z is the region in the left of the perpendicular bisector of the join of 0 and -1.



Question 7

(a) $\partial V = 2\pi Rh\partial x = 2\pi(1-x)y\partial x$

$= 2\pi\frac{(1-x)x}{1+x^2}\partial x = 2\pi\frac{x-x^2}{1+x^2}\partial x$

$= 2\pi\left(-1+\frac{x+1}{1+x^2}\right)\partial x$

$\therefore V = 2\pi\int_0^1\left(-1+\frac{x+1}{1+x^2}\right)dx$

$= 2\pi\left[-x+\frac{1}{2}\ln(1+x^2)+\tan^{-1}x\right]_0^1$

$= 2\pi\left(-1+\frac{1}{2}\ln 2+\frac{\pi}{4}\right)$ u^3.

(b) (i) Let $u = 4-x, du = -dx$.

When $x=1, u=3$; when $x=3, u=1$.

$$\int_1^3\frac{\cos^2\left(\frac{\pi}{8}x\right)}{x(4-x)}dx = \int_3^1\frac{\cos^2\left(\frac{\pi}{8}(4-u)\right)}{u(4-u)}(-du)$$

$$= \int_1^3\frac{\cos^2\left(\frac{\pi}{2}-\frac{\pi}{8}u\right)}{u(4-u)}du = \int_1^3\frac{\sin^2\left(\frac{\pi}{8}u\right)}{u(4-u)}du$$

(ii) $\therefore I = \int_1^3\frac{\cos^2\left(\frac{\pi}{8}x\right)}{x(4-x)}dx = \int_1^3\frac{\sin^2\left(\frac{\pi}{8}x\right)}{x(4-x)}dx.$

$$\therefore I = \frac{1}{2}\int_1^3\frac{\cos^2\left(\frac{\pi}{8}x\right)+\sin^2\left(\frac{\pi}{8}x\right)}{x(4-x)}dx$$

$$= \frac{1}{2}\int_1^3\frac{1}{x(4-x)}dx = \frac{1}{8}\int_1^3\left(\frac{1}{x}+\frac{1}{4-x}\right)dx$$

$$= \frac{1}{8}\left[\ln\frac{x}{4-x}\right]_1^3 = \frac{1}{8}\ln\frac{3}{\frac{1}{3}} = \frac{1}{8}\ln 9.$$

(c) (i) $\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2} = 1.$

$\left(b^2+a^2m^2\right)x^2+2a^2mcx+\left(a^2c^2-a^2b^2\right) = 0.$

The line is a tangent, $\therefore \Delta = 0.$

$4a^4m^2c^2-4\left(b^2+a^2m^2\right)\left(a^2c^2-a^2b^2\right) = 0.$

$a^2m^2c^2-\left(b^2+a^2m^2\right)\left(c^2-b^2\right) = 0$

$a^2m^2c^2-b^2c^2+b^4-a^2m^2c^2+a^2m^2b^2 = 0$

$-c^2+b^2+a^2m^2 = 0$

$\therefore b^2+a^2m^2 = c^2.$

(ii) The equation of ℓ is $mx-y+c=0$,

$$\therefore QS = \left|\frac{mae-0+c}{\sqrt{m^2+1}}\right| = \left|\frac{mae+c}{\sqrt{m^2+1}}\right|$$

(iii) $QS \times Q'S' = \left|\frac{mae+c}{\sqrt{m^2+1}}\right|\left|\frac{mae-c}{\sqrt{m^2+1}}\right|$

$$= \left|\frac{m^2a^2e^2-c^2}{m^2+1}\right| = \left|\frac{m^2a^2e^2-(b^2+a^2m^2)}{m^2+1}\right|$$

$$= \left|\frac{m^2a^2\left(e^2-1\right)-b^2}{m^2+1}\right| = \left|\frac{-m^2b^2-b^2}{m^2+1}\right|$$

$$= b^2\left|\frac{m^2+1}{m^2+1}\right| = b^2.$$

Question 8

(a) $I_m=\int_0^1 x^m(x^2-1)^5dx$

Let $u=x^{m-1}, du=(m-1)x^{m-2}$

and $dv=x(x^2-1)^5dx, v=\frac{(x^2-1)^6}{12}$.

$\therefore I_m=\left[x^{m-1}\frac{(x^2-1)^6}{12}\right]_0^1-\frac{m-1}{12}\int_0^1 x^{m-2}(x^2-1)^6dx$

$=-\frac{m-1}{12}\int_0^1 x^{m-2}(x^2-1)(x^2-1)^5dx$

$=-\frac{m-1}{12}\int_0^1 (x^m-x^{m-2})(x^2-1)^5dx$

$=-\frac{m-1}{12}(I_m-I_{m-2})$

$\therefore (12+m-1)I_m=(m-1)I_{m-2}$.

$\therefore I_m=\frac{m-1}{m+11}I_{m-2}$.

(b) (i) $\frac{7}{7}\times\frac{6}{7}\times\ldots\times\frac{1}{7}=\frac{7!}{7^7}$.

(ii) $1-\text{Pr(all balls are selected)}=1-\frac{7!}{7^7}$.

(iii) Choosing a ball not to be selected in ${}^7C_1$ ways, choosing a ball to repeat in ${}^6C_1$ ways, arranging the 7 balls, of which 2 are the same in $\frac{7!}{2!}$ ways, and each ball has $\frac{1}{7}$ chance of being selected, $\therefore$ Pr(exactly one ball is not selected)

$={}^7C_1\times{}^6C_1\times\frac{7!}{2!}\times\left(\frac{1}{7}\right)^7$.

(c) (i) Since β is a root,

$\beta^n+a_{n-1}\beta^{n-1}+\ldots+a_1\beta+a_0=0$.

$\therefore \beta^n=-a_{n-1}\beta^{n-1}-\ldots-a_1\beta-a_0$.

$\therefore |\beta^n|=|a_{n-1}\beta^{n-1}+\ldots+a_1\beta+a_0|$

$\le|a_{n-1}\beta^{n-1}|+\ldots+|a_1\beta|+|a_0|$, since the modulus of the sum of n complex numbers is always less than the sum of the moduli of these n complex numbers (triangular inequality is a special case of 2 numbers)

$\le M\left(|\beta^{n-1}|+\ldots+|\beta|+1\right)$, since $a_{n-1},\ldots,a_1,a_0$ all are less than or equal M.

(ii) Inside the brackets is a GP, its sum $=\frac{|\beta|^n-1}{|\beta|-1}$.

$|\beta|^n\le M\left(\frac{|\beta|^n-1}{|\beta|-1}\right)=M\left|\frac{|\beta|^n-1}{|\beta|-1}\right|$, noting that $|\beta^{n-1}|+\ldots+|\beta|+1>0$ always.

$1\le M\left|\frac{1-\frac{1}{|\beta|^n}}{|\beta|-1}\right|$, on dividing by $|\beta|^n$

$1<M\left|\frac{1}{|\beta|-1}\right|$.

$\therefore |\beta|-1<M$, or $|\beta|<1+M$.

(d) Let β be the root of the equation $S(z)=\sum_{k=0}^{n}c_kz^k$, where $z=x+\frac{1}{x}$, $\therefore\beta$ is also the root of $\sum_{k=0}^{n}\frac{c_k}{c_n}z^k$ (so that the coefficient of z^n is 1)

From part (c), $|\beta|<1+M$, where M is the maximum of $\left|\frac{c_k}{c_n}\right|$, but $|c_k|\le|c_n|, \therefore M\le 1$.

$\therefore |\beta|<2, \therefore \left|x+\frac{1}{x}\right|<2$.

$x^2-2x+1<0$.

$(x-1)^2<0$.

$\therefore x$ is not real.