

Question 1

$$\begin{aligned} \text{(a)} \int \frac{x}{\sqrt{9-4x^2}} dx &= -\frac{1}{8} \int \frac{-8x}{\sqrt{9-4x^2}} dx \\ &= -\frac{1}{8} \cdot 2\sqrt{9-4x^2} + C \\ &= -\frac{\sqrt{9-4x^2}}{4} + C. \end{aligned}$$

$$\begin{aligned} \text{(b)} \int \frac{dx}{x^2-6x+13} &= \int \frac{1}{(x-3)^2+4} dx \\ &= \frac{1}{2} \tan^{-1} \frac{x-3}{2} + C. \end{aligned}$$

$$\begin{aligned} \text{(c)} a = \lim_{x \rightarrow 3} \frac{16x-43}{x+2} &= \frac{5}{5} = 1, c = \lim_{x \rightarrow -2} \frac{16x-43}{(x-3)^2} = \frac{-75}{25} = -3 \\ b = 3, & \text{ by equating the coefficients of } x^2. \end{aligned}$$

$$\begin{aligned} \int \frac{16x-43}{(x-3)^2(x+2)} dx &= \int \left(\frac{1}{(x-3)^2} + \frac{3}{x-3} - \frac{3}{x+2} \right) dx \\ &= \frac{-1}{x-3} + 3\ln|x-3| - 3\ln|x+2| + C. \end{aligned}$$

(d) Let $u = t, dv = e^{-t} dt$ then $du = dt, v = -e^{-t}$.

$$\begin{aligned} \int_0^2 te^{-t} dt &= [-te^{-t}]_0^2 + \int_0^2 e^{-t} dt \\ &= -2e^{-2} - [e^{-t}]_0^2 \\ &= -2e^{-2} - e^{-2} + 1 \\ &= -3e^{-2} + 1. \end{aligned}$$

(e) Let $t = \tan \frac{\theta}{2}, dt = \frac{1}{2} \sec^2 \frac{\theta}{2} d\theta, \therefore d\theta = \frac{2dt}{1+t^2}$.

When $\theta = \frac{\pi}{2}, t = 1$. When $\theta = \frac{2\pi}{3}, t = \tan \frac{\pi}{3} = \sqrt{3}$.

$$\begin{aligned} \int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \frac{1}{\sin \theta} dx &= \int_1^{\sqrt{3}} \frac{1+t^2}{2t} \frac{2dt}{1+t^2} \\ &= \int_1^{\sqrt{3}} \frac{1}{t} dt \\ &= [\ln t]_1^{\sqrt{3}} \\ &= \ln \sqrt{3}. \end{aligned}$$

Question 2

(a) (i) $z^2 = (3+i)^2 = 9-1+6i = 8+6i$.

(ii) $\bar{z}w = (3-i)(2-5i) = 6-5-2i-15i = 1-17i$.

(iii) $\frac{w}{z} = \frac{2-5i}{3+i} = \frac{(2-5i)(3-i)}{(3+i)(3-i)} = \frac{1-17i}{10}$.

(b) (i) $\sqrt{3}-i = 2 \operatorname{cis} \left(-\frac{\pi}{6} \right)$.

(ii) $(\sqrt{3}-i)^7 = 128 \operatorname{cis} \frac{-7\pi}{6}$
 $= 128 \operatorname{cis} \frac{5\pi}{6}$

(iii) $(\sqrt{3}-i)^7 = 64(-\sqrt{3}+i)$.

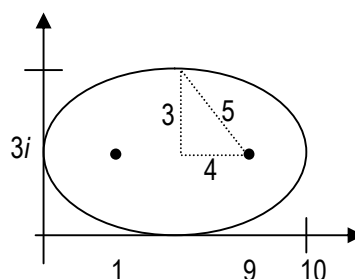
(c) $-1 = \operatorname{cis}(\pi + 2k\pi)$.

$$\begin{aligned} z = \sqrt[3]{-1} &= \operatorname{cis} \frac{\pi + 2k\pi}{3}, k = 0, \pm 1, \\ &= \operatorname{cis} \frac{\pi}{3}, \operatorname{cis} \left(-\frac{\pi}{3} \right), \operatorname{cis} \pi = -1. \end{aligned}$$

(d) (i) $5+3i$.

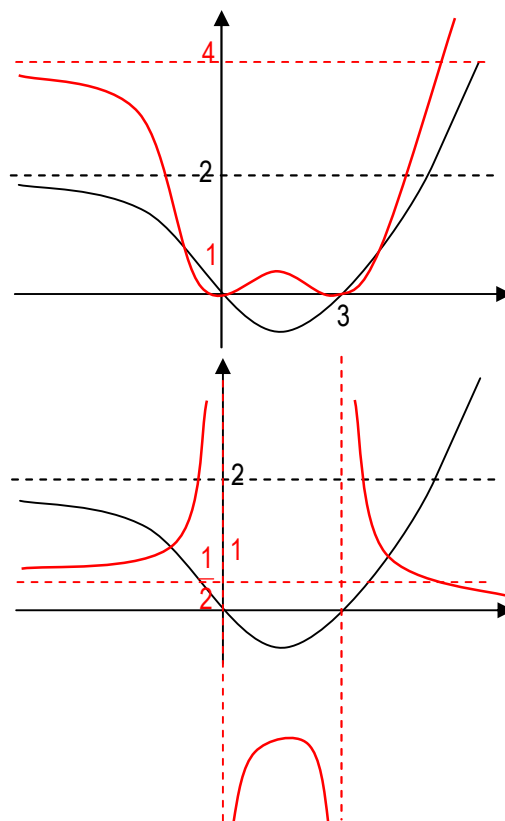
(ii) major axis = 10, minor axis = 6.

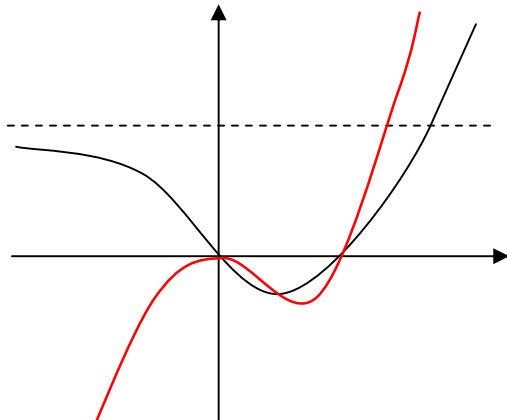
(iii) $0 \leq \arg(z) \leq \frac{\pi}{2}$.



Question 3

(a)





- (b) (i) $y = 0, x^2 = 144, \therefore x = \pm 12$.
 (ii) $\pm(\sqrt{144 + 25}, 0) = \pm(13, 0)$.
 (iii) directrices $x = \pm \frac{144}{13}$.
 asymptotes $y = \pm \frac{5x}{12}$.

- (c) (i) $\sum \alpha = 4a = 12, \therefore a = 3$.
 $\prod \alpha = (a^2 + b^2)(a^2 + 4b^2) = 130$.
 $(9 + b^2)(9 + 4b^2) = 130$.
 $4b^4 + 45b^2 - 49 = 0$.
 $(4b^2 + 49)(b^2 - 1) = 0$.
 $\therefore b^2 = 1$ (b is real)
 $\therefore b = 1$.
 (ii) $P(x) = (x - 3 + i)(x - 3 - i)(x - 3 + 2i)(x - 3 - 2i)$
 $= ((x - 3)^2 + 1)((x - 3)^2 + 4)$
 $= (x^2 - 6x + 10)(x^2 - 6x + 13)$.

Question 4

- (a) $p'(x) = 3ax^2 + b$.
 $p'(1) = 0$ gives $3a + b = 0$. (1)
 $p(1) = 0$ gives $a + b + c = 0$. (2)
 $p(-1) = 4$ gives $-a - b + c = 4$. (3)
 (2) + (3) gives $2c = 4, \therefore c = 2$.
 (1) + (3) gives $2a + 2 = 4, \therefore a = 1$.
 (1) gives $b = -3$.

- (b) Side = $2x$.
 Area = $4x^2$.
 $\partial V = 4x^2 \partial y$.
 $V = \int_0^1 4x^2 dy = \int_0^1 4y dy$
 $= [2y^2]_0^1 = 2 \text{ units}^3$.

- (c) (i) $m_1 = \frac{p}{p-q} = -\frac{1}{pq}, \therefore m_2 = pq$.

$$y - \frac{1}{r} = pq(x - r)$$

$$y = pqx - pqr + \frac{1}{r}. \quad (1)$$

$$(ii) y = qrx - pqr + \frac{1}{p}. \quad (2)$$

(iii) (1) - (2) gives

$$0 = q(p-r)x + \frac{1}{r} - \frac{1}{p}$$

$$0 = q(p-r)x + \frac{p-r}{pr}$$

$$0 = qx + \frac{1}{pr}$$

$$\therefore x = -\frac{1}{pqr}$$

$$\text{Put to (1), } y = pqx - pqr + \frac{1}{r}$$

$$= -\frac{1}{r} - pqr + \frac{1}{r}$$

$$= -pqr.$$

Since $xy = \left(-\frac{1}{pqr}\right)(-pqr) = 1$, this point belongs

to the hyperbola.

- (d) (i) $KM \parallel BL$ and $KM = BL$ (the join of the midpoints of two sides of a triangle is parallel to and equal half the third side.)
 (ii) the interior angle in cyclic quad $KMLP$ is equal to the opposite exterior angle. Let $\angle KML$ be α .
 (ii) $KB = KA$ (K is midpoint of AB)
 $KP = KB$ ($\angle KBP = \angle KPB = \alpha, \therefore$ isos Δ)
 $\therefore KP = KA$.
 $\therefore \angle KPA = \angle KAP$. Let it be β .
 $\therefore \angle KPB + \angle KPA = \alpha + \beta$.
 But $2\alpha + 2\beta = 180^\circ$ (angle sum in ΔABP)
 $\therefore \angle KPB + \angle KPA = 90^\circ, \therefore AP \perp BP$.

Question 5

- (a) $\partial V = 2\pi xy \partial x$

$$V = 2\pi \int_0^1 x^2(x-1)^2 dx$$

$$= 2\pi \int_0^1 (x^4 - 2x^3 + x^2) dx$$

$$= 2\pi \left[\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} \right]_0^1$$

$$= \frac{\pi}{15} \text{ units}^3.$$

$$(b) (i) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta. \quad (1)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta. \quad (2)$$

(1) + (2) gives

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta.$$

$$(ii) \cos \theta + \cos 3\theta + \cos 2\theta + \cos 4\theta$$

$$= 2 \cos \theta \cos 2\theta + 2 \cos \theta \cos 3\theta$$

$$= 2 \cos \theta (\cos 2\theta + \cos 3\theta)$$

$$= 2 \cos \theta \cdot 2 \cos \frac{\theta}{2} \cos \frac{5\theta}{2}.$$

$$\text{Solve } 2 \cos \theta \cdot 2 \cos \frac{\theta}{2} \cos \frac{5\theta}{2} = 0 \text{ gives}$$

$$\cos \theta = 0, \cos \frac{\theta}{2} = 0, \cos \frac{5\theta}{2} = 0.$$

$$\therefore \theta = \frac{\pi}{2} + k\pi, \frac{\theta}{2} = \frac{\pi}{2} + k\pi, \frac{5\theta}{2} = \frac{\pi}{2} + k\pi.$$

$$\therefore \theta = \frac{\pi}{2} + k\pi, \theta = \pi + 2k\pi, \theta = \frac{\pi}{5} + \frac{2k\pi}{5}.$$

$$\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}.$$

$$(c) (i) \text{ vert, } T_1 \cos \alpha = T_2 \cos \alpha + mg. \quad (1)$$

$$\text{hor, } T_1 \sin \alpha + T_2 \sin \alpha = ml \sin \alpha \omega^2. \quad (2)$$

(ii) Put $T_2 = 0$,

$$\frac{(2)}{(1)} \text{ gives } \tan \alpha = \frac{ml \sin \alpha \omega^2}{mg}.$$

$$\therefore \omega^2 = \frac{g}{l \cos \alpha}.$$

$$(d) (i) 3^4.$$

$$(ii) 0.2 \times 0.6 \times 0.2 \times 0.6 = 0.0144.$$

$$(iii) Pr = 4W + 3W + 2W2D + 2W1D1L + 1W3D$$

$$= 0.2^4 + {}^4C_3 \cdot 0.2^3 \times 0.8 + \frac{4!}{2!2!} \cdot 0.2^2 \times 0.6^2$$

$$+ \frac{4!}{2!} \cdot 0.2^2 \times 0.6 \times 0.2 + \frac{4!}{3!} \cdot 0.6^3 \times 0.2$$

$$= 0.344.$$

Question 6

(a) (i) Using the Sine rule,

$$\frac{\sin \theta}{OB} = \frac{\sin(\beta - \theta)}{OA}.$$

$$\therefore \frac{OA}{OB} = \frac{\sin(\beta - \theta)}{\sin \theta}.$$

$$(ii) \text{ Similarly, } \frac{OB}{OC} = \frac{\sin(\gamma - \theta)}{\sin \theta}, \frac{OC}{OA} = \frac{\sin(\alpha - \theta)}{\sin \theta}.$$

Multiplying all the three,

$$\frac{OA}{OB} \cdot \frac{OB}{OC} \cdot \frac{OC}{OA} = \frac{\sin(\beta - \theta)}{\sin \theta} \cdot \frac{\sin(\gamma - \theta)}{\sin \theta} \cdot \frac{\sin(\alpha - \theta)}{\sin \theta}.$$

$$\therefore \sin^3 \theta = \sin(\beta - \theta) \sin(\gamma - \theta) \sin(\alpha - \theta).$$

$$(iii) \cot x - \cot y = \frac{\cos x}{\sin x} - \frac{\cos y}{\sin y}$$

$$= \frac{\sin y \cos x - \cos y \sin x}{\sin x \sin y}$$

$$= \frac{\sin(y - x)}{\sin x \sin y}.$$

$$(iv) (\cot \theta - \cot \alpha)(\cot \theta - \cot \beta)(\cot \theta - \cot \gamma)$$

$$= \frac{\sin(\theta - \alpha)}{\sin \theta \sin \alpha} \frac{\sin(\theta - \beta)}{\sin \theta \sin \beta} \frac{\sin(\theta - \gamma)}{\sin \theta \sin \gamma}$$

$$= \frac{\sin^3 \theta}{\sin \theta \sin \alpha \sin \theta \sin \beta \sin \theta \sin \gamma}, \text{ from (ii)}$$

$$= \text{cosec } \alpha \text{ cosec } \beta \text{ cosec } \gamma.$$

$$(v) \text{ If } \alpha = \frac{\pi}{2}, \beta = \gamma = \frac{\pi}{4},$$

$$\left(\cot \theta - \cot \frac{\pi}{2} \right) \left(\cot \theta - \cot \frac{\pi}{4} \right) \left(\cot \theta - \cot \frac{\pi}{4} \right)$$

$$= \text{cosec } \frac{\pi}{2} \text{ cosec } \frac{\pi}{4} \text{ cosec } \frac{\pi}{4}.$$

$$\cot \theta (\cot \theta - 1)^2 = 2.$$

$$\cot^3 \theta - 2 \cot^2 \theta + \cot \theta - 2 = 0.$$

$$(\cot \theta - 2)(\cot^2 \theta) + \cot \theta - 2 = 0$$

$$(\cot \theta - 2)(\cot^2 \theta + 1) = 0.$$

$$\cot \theta = 2.$$

$$\theta = \tan^{-1} \frac{1}{2}.$$

(b) (i) On the planet's surface, $\ddot{x} = -g$.

$$-g = -\frac{k}{R^3}.$$

$$\therefore k = gR^3.$$

$$(ii) \ddot{x} = \frac{v dv}{dx} = -\frac{gR^3}{x^3}.$$

$$\int v dv = -gR^3 \int \frac{1}{x^3} dx$$

$$\frac{v^2}{2} = gR^3 \cdot \frac{1}{2x^2} + C.$$

$$\text{When } x = R, v = u, \therefore C = \frac{u^2}{2} - \frac{gR}{2}.$$

$$\therefore v^2 = \frac{gR^3}{x^2} + u^2 - gR.$$

$$= \frac{gR^3}{x^2} - (gR - u^2).$$

$$(iii) \text{ If } u^2 \geq gR \text{ then } v^2 \geq \frac{gR^3}{x^2}.$$

$$v^2 \geq \frac{gR^3}{R^2 + 3uRt - (gR - u^2)t^2}.$$

As $t \rightarrow \infty, v^2 \rightarrow 0$: the particle will not return.

$$(iv) (1) \text{ When } v = 0, \frac{gR^3}{x^2} = (gR - u^2),$$

$$\therefore x^2 = \frac{gR^3}{gR - u^2}.$$

$$\therefore D = \sqrt{\frac{gR^3}{gR - u^2}}.$$

$$(2) \left(\frac{gR^3}{gR - u^2} \right) = R^2 + 2uRt - (gR - u^2)t^2$$

$$(gR - u^2)t^2 - 2uRt + R^2 - \left(\frac{gR^3}{gR - u^2} \right) = 0.$$

$$t = \frac{uR + \sqrt{u^2R^2 - (gR - u^2) \left(R^2 - \left(\frac{gR^3}{gR - u^2} \right) \right)}}{(gR - u^2)}$$

$$= \frac{uR}{(gR - u^2)}.$$

Question 7

$$(a) (i) \cos x = \tan x$$

$$= \frac{\sin x}{\cos x}.$$

$$\cos^2 x - \sin x = 0.$$

If $x = \alpha$ is the point of intersection, then

$$\cos^2 \alpha - \sin \alpha = 0. \quad (1)$$

$$\text{For } y = \cos x, \frac{dy}{dx} = -\sin x. \therefore m_1 = -\sin \alpha.$$

$$\text{For } y = \tan x, \frac{dy}{dx} = \sec^2 x. \therefore m_2 = \sec^2 \alpha = \frac{1}{\cos^2 \alpha}.$$

$$\text{But } \cos^2 \alpha = \sin \alpha.$$

$$\therefore m_1 m_2 = -\sin \alpha \cdot \frac{1}{\cos^2 \alpha} = -1.$$

\therefore The curves intersect at right angles at $x = \alpha$.

(ii) From (1),

$$1 - \sin^2 \alpha - \sin \alpha = 0.$$

$$\sin^2 \alpha + \sin \alpha - 1 = 0.$$

$$\sin \alpha = \frac{-1 + \sqrt{5}}{2} \text{ (taking the + value for } 0 < \alpha < \frac{\pi}{2} \text{.)}$$

$$\tan \alpha = \frac{-1 + \sqrt{5}}{\sqrt{4 - (-1 + \sqrt{5})^2}}$$

$$= \frac{-1 + \sqrt{5}}{\sqrt{4 - 1 + 2\sqrt{5} - 5}}$$

$$= \frac{-1 + \sqrt{5}}{\sqrt{2(-1 + \sqrt{5})}} = \sqrt{\frac{-1 + \sqrt{5}}{2}}.$$

$$\sec^2 \alpha = 1 + \tan^2 \alpha = 1 + \frac{-1 + \sqrt{5}}{2}$$

$$= \frac{1 + \sqrt{5}}{2}.$$

$$(b) (i) I_n = \int_0^x \sec^n t \, dt.$$

$$\text{Let } u = \sec^{n-2} t, dv = \sec^2 t \, dt.$$

$$du = (n-2) \sec^{n-2} t \cdot \tan t \, dt, v = \tan t.$$

$$I_n = \left[\sec^{n-2} t \tan t \right]_0^x - (n-2) \int_0^x \sec^{n-2} t \tan^2 t \, dt$$

$$= \sec^{n-2} x \tan x - (n-2) \int_0^x \sec^{n-2} t (\sec^2 t - 1) \, dt$$

$$= \sec^{n-2} x \tan x - (n-2) I_n + (n-2) I_{n-2}.$$

$$\therefore (n-1) I_n = \sec^{n-2} x \tan x + (n-2) I_{n-2}.$$

$$\therefore I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2}.$$

$$(ii) I_4 = \frac{\sec^2 \frac{\pi}{3} \tan \frac{\pi}{3}}{3} + \frac{2}{3} I_2$$

$$= \frac{4\sqrt{3}}{3} + \frac{2}{3} I_2.$$

$$I_2 = \tan \frac{\pi}{3} + 0 = \sqrt{3}.$$

$$\therefore I_4 = \frac{4\sqrt{3}}{3} + \frac{2\sqrt{3}}{3} = 2\sqrt{3}.$$

$$(c) \text{ When } n=1, x_1 = 2 \left(\frac{1+\alpha}{1-\alpha} \right)$$

$$= 2 \left(\frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \right)$$

$$= 2 \left(\frac{2}{4} \right)$$

$$= 1.$$

True for $n=1$.

Assume $x_n = 2 \left(\frac{1+\alpha^n}{1-\alpha^n} \right)$ is true, we are required to

prove that it's true for x_{n+1} .

$$x_{n+1} = \frac{4 + x_n}{1 + x_n}$$

$$= \frac{4 + 2 \left(\frac{1+\alpha^n}{1-\alpha^n} \right)}{1 + 2 \left(\frac{1+\alpha^n}{1-\alpha^n} \right)}$$

$$= \frac{4 + 2 \left(\frac{1+\alpha^n}{1-\alpha^n} \right)}{1 + 2 \left(\frac{1+\alpha^n}{1-\alpha^n} \right)}$$

$$\begin{aligned}
&= \frac{4 - 4\alpha^n + 2 + 2\alpha^n}{1 - \alpha^n + 2 + 2\alpha^n} \\
&= 2 \left(\frac{3 - \alpha^n}{3 + \alpha^n} \right) \\
&= 2 \left(\frac{1 - \alpha^n \cdot \frac{1}{3}}{1 + \alpha^n \cdot \frac{1}{3}} \right) \\
&= 2 \left(\frac{1 + \alpha^{n+1}}{1 - \alpha^{n+1}} \right).
\end{aligned}$$

$\therefore x_{n+1}$ is true.

\therefore It's true for all $n \geq 1$.

(ii) As $n \rightarrow \infty, \alpha^n \rightarrow 0, \therefore x_n \rightarrow 2$.

Question 8

(a) (i) $t \leq \frac{1}{\sqrt{2}}$.

$$t^2 \leq \frac{1}{2}.$$

$$1 - t^2 \geq 1 - \frac{1}{2} = \frac{1}{2}.$$

$$\frac{1}{1 - t^2} \leq 2.$$

$$\therefore \frac{2t^2}{1 - t^2} \leq 4t^2.$$

Also, since $t \leq \frac{1}{\sqrt{2}}, 1 - t^2 \geq 0, \therefore \frac{2t^2}{1 - t^2} \geq 0$.

(ii) By long division, $\frac{2t^2}{1 - t^2} = -2 + \frac{2}{1 - t^2}$

then by partial fractions, $= -2 + \frac{1}{1 + t} + \frac{1}{1 - t}$.

$$\therefore 0 \leq -2 + \frac{1}{1 + t} + \frac{1}{1 - t} \leq 4t^2.$$

(iii) $0 \leq \int_0^x \left(-2 + \frac{1}{1 + t} + \frac{1}{1 - t} \right) dt \leq \int_0^x 4t^2 dt, 0 \leq x \leq \frac{1}{\sqrt{2}}$.

$$\left[-2t + \ln(1 + t) - \ln(1 - t) \right]_0^x \leq \left[\frac{4t^3}{3} \right]_0^x.$$

$$-2x + \ln \frac{1 + x}{1 - x} \leq \frac{4x^3}{3}.$$

(iv) $\ln e^{-2x} + \ln \frac{1 + x}{1 - x} \leq \ln e^{\frac{4x^3}{3}}$.

$$\ln \left(\frac{1 + x}{1 - x} \right) e^{-2x} \leq \ln e^{\frac{4x^3}{3}}.$$

$$\therefore \left(\frac{1 + x}{1 - x} \right) e^{-2x} \leq e^{\frac{4x^3}{3}}.$$

(b) (i) $f'(x) = nx^{n-1}e^{-x} - x^n e^{-x}$

$$= x^{n-1}(n - x)e^{-x}.$$

$$f''(x) = n(n-1)x^{n-2}e^{-x} - nx^{n-1}e^{-x} - x^{n-1}(n-x)e^{-x}$$

$$= x^{n-2}e^{-x} (n(n-1) - nx - x(n-x))$$

$$= x^{n-2}e^{-x} (x^2 - 2nx + n(n-1)).$$

$$f''(x) = 0 \text{ gives } x = n \pm \sqrt{n^2 - n(n-1)}$$

$$= n \pm \sqrt{n}.$$

$f''(x)$ changes signs at these points so they are points of inflexion.

$$\therefore b = n + \sqrt{n}, a = n - \sqrt{n} \text{ (so that } b > a).$$

(ii) $\frac{f(b)}{f(a)} = \frac{(n + \sqrt{n})^n e^{-(n + \sqrt{n})}}{(n - \sqrt{n})^n e^{-(n - \sqrt{n})}}$

$$= \left(\frac{n + \sqrt{n}}{n - \sqrt{n}} \right)^n e^{-2\sqrt{n}}$$

$$= \left(\frac{1 + \frac{\sqrt{n}}{n}}{1 - \frac{\sqrt{n}}{n}} \right)^n e^{-2\sqrt{n}} = \left(\frac{1 + \frac{1}{\sqrt{n}}}{1 - \frac{1}{\sqrt{n}}} \right)^n e^{-2\sqrt{n}}.$$

(iii) Replacing x by $\frac{1}{\sqrt{n}}$

$$\left(\frac{1 + x}{1 - x} \right) e^{-2x} \leq e^{\frac{4x^3}{3}} \text{ becomes } \left(\frac{1 + \frac{1}{\sqrt{n}}}{1 - \frac{1}{\sqrt{n}}} \right) e^{\frac{-2}{\sqrt{n}}} \leq e^{\frac{4}{3n\sqrt{n}}}.$$

$$\therefore \left(\frac{1 + \frac{1}{\sqrt{n}}}{1 - \frac{1}{\sqrt{n}}} \right)^n e^{-2\sqrt{n}} = \left(\left(\frac{1 + \frac{1}{\sqrt{n}}}{1 - \frac{1}{\sqrt{n}}} \right) e^{\frac{-2}{\sqrt{n}}} \right)^n \leq \left(e^{\frac{4}{3n\sqrt{n}}} \right)^n$$

$$= e^{\frac{4n}{3n\sqrt{n}}}$$

$$= e^{\frac{4}{3\sqrt{n}}}.$$

$$\therefore \frac{f(b)}{f(a)} \leq e^{\frac{4}{3\sqrt{n}}}.$$

$\frac{f(b)}{f(a)} \geq 1$ because the numerator is always more than

the denominator.

$$\therefore 1 \leq \frac{f(b)}{f(a)} \leq e^{\frac{4}{3\sqrt{n}}}.$$

(iv) As $n \rightarrow \infty, e^{\frac{4}{3\sqrt{n}}} \rightarrow 1, \therefore \frac{f(b)}{f(a)} = 1$.